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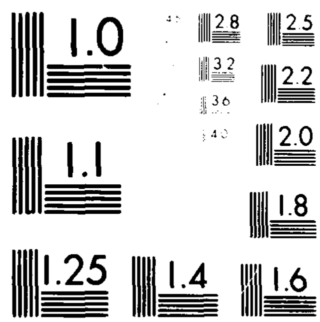


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# EXPERIMENTAL RANDOM FATIGUE IN ELASTIC RANGE - MODELS OF SIGNIFICANT VARIABLES


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EXPERIMENTAL RANDOM FATIGUE IN ELASTIC RANGE-MODELS OF

SIGNIFICANT VARIABLES

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Abstract

In the previous study of first order models with 8 variables to predict fatigue life under random vibrations, 4 variables showed significant effects. In this report both first and second order models of these 4 significant variables based on 10, 18, and 24 tests are developed. The tables of analysis of variance, and of predicted lives together with their residuals and 95% confidence intervals are constructed for each of the first and second order models. A second order model of 4 significant variables consisting of 14 terms, based on 24 tests, has been found to be the statistically best one. The deviations of the lives predicted by this model range from -26.6% to 20.4% with an average of 5.5% on the negative side and 7.0% on the positive side. These results are in contrast with those which are obtained by the principle of linear damage accumulation based on cycle counting and involve several hundred percent error.

## INTRODUCTION

In the previous report [1] it was noted that the principle of linear damage accumulation based on equivalent cycle counting gives unreliable estimates of fatigue life under random vibrations [2-11]. In this group of selected references [2-11], the principle of linear damage accumulation was applied and the fatigue life was found to be overestimated by a factor ranging from -5.0 to 15.0 in [2-9]. In [10,11] it was reported that the fatigue life was overestimated but the factor of overestimation was not given.

A new history-dependent stochastic model of cumulative damage is being developed by Bogdanoff [12,13,14] by taking a comprehensive view of entire failure process to improve the predictive accuracy. Another phenomenological approach entirely different from that of all the cumulative damage approaches mentioned above was undertaken in the previous study [1] to develop first order models or life predicting equations. In this approach a test program based on 8 probabilistic parameters and experiment design was conducted in 3 designs. The first order models are developed for each design and the four variables which showed significant effects on the fatigue life are considered in the present report. Both the first order and second order models of the significant variables are developed for the same three designs as in [1]. From a comparison of all the models considered the statistically best one is singled out.

Considering the models of significant variables is essential in our search for the statistically best model. The other models, which consist of terms of any combination of insignificant variables are inferior because the number of degrees of freedom of regression

increases with a negligible increase in the regression sum of squares. Simultaneously, the confidence intervals are widened depending upon the number of degrees of freedom added to regression. Therefore such models need not be considered.

## I. EXPERIMENTS, PARAMETERS, DESIGNS AND MODELS

The mean, variance, duration of excursion above $\varepsilon_f$ level, and the band width were found to be 4 significant variables on the basis of the analysis in the previous report [1] of the first order models of all 8 variables. In the present report these 4 variables are considered for both first and second order models. In addition, a model with all second order terms of significant variables and first order terms of all 8 variables is also constructed. All these models are expected to improve the prediction of fatigue life under random vibrations over the first order models in [1].

In this report 3 designs which were used in [1] will be considered. For the first design, a full factorial design consisting of 10 tests, only first order model of significant variables could be constructed because a minimum of 15 tests are required for a second order model based on 4 variables. For the second design, a central composite design of 18 tests including 4 center points, first order and second order models using only the significant variables are developed. For the third design, a central composit design of 24 tests including 4 center points and 6 replications, three models are developed. The first two are the first and second order models involving the four significant variables only as was done for the second design. The third model involves first order terms of all 8 variables and all second order terms of significant variables. A minimum of 19 tests are

required to develop this model. For this reason, the third model could not be developed for the first and second designs.

For each of these three designs the tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals are constructed. The F-ratio was computed for each model to check if the model is acceptable. The confidence intervals are computed from the standard deviations of the predicted lives and the t values from the t-table corresponding to the degrees of freedom of the residuals of the model. The distribution of residuals was studied for any trend or pattern present. In case of any trend or pattern present the model was considered inadequate.

Finally a comparison of all models is made.

## II. FULL FACTORIAL DESIGN WITH TWO CENTER POINTS

### 1. First Order Model of 4 Significant Variables

A first order model is obtained using the first ten tests of the experiment by regressing log of fatigue life, y, on the coded variables under consideration. These 10 tests form a full factorial design with 2 center points. The first order life predicting equation is obtained as

$$\hat{y} = 5.87 - 0.094x_1 - 0.782x_2 - 0.0510x_6 - 0.100x_7 \quad (1)$$

The analysis of variance is given in Table 1. The F-ratio for this equation is 16.91 which is greater than the corresponding F value of 5.19 from F-table with 4 and 5 degrees of freedom at 95% significance level. So the regression is effective and the model is acceptable. The residual sum of squares is 0.3983 in comparison to a total of 5.788, a 6.9%. The other 93.1% of the total is due to regression.

The sum of squares due to duration of excursion above $\varepsilon_f$ level is fairly low in comparison to sum of squares due to other 3 variables. This variable will be considered in the remaining two designs based on high number of tests in order to further investigate its individual effects in the first order models, and its individual and interaction effects in the second order models.

Table 2 gives the lives predicted by Eq. (1) together with their residuals and 95% confidence intervals. All the actual lives fall within the predicted confidence intervals. These confidence intervals are wide because the number of observations is relatively small. On the basis of the 10 tests, it will not be very meaningful to draw any conclusion on the basis of the distribution of residuals.

2. Second Order Model of 4 Significant Variables

For the first 10 tests only the first order model could be developed for this design because the number of tests are not sufficient to develop the second order model.

## III. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS

This design consists of 18 tests. Two life predicting equations, a first order and a second order, are obtained for this design which are given below.

1. First order model of 4 Significant Variables

The first order life predicting equation is obtained as

$$\hat{y} = 5.84 - 0.039x_1 - 0.708x_2 - 0.080x_6 - 0.108x_7 \qquad (2)$$

The analysis of variance of this equation is given in Table 3. The F-ratio is computed to be 33.86 with 4 and 13 degrees of freedom. The corresponding F value from the F-table is 3.18 at 95% significance level. The comparison of the two F values shows that the regression

is effective and the model is acceptable. The residual sum of squares is 0.8568 in comparison to a total of 9.7843, a 8.8%. The other 91.2% of the total is due to regression. The sum of squares contributed due to the duration of excursion above $\varepsilon_f$ level, in this case also, is comparatively low, which implies that this variable may not have a significant effect on the fatigue life. In order to verify this fact, this variable will be considered for the analysis of 24 tests which is described in the third design.

The predicted lives together with their residuals and 95 percent confidence intervals are given in Table 4. The confidence intervals cover all the actual lives of the tests except for the test no. 4. The lower limit of the confidence interval of test 4 is slightly above its actual life. That the residuals appear to be randomly distributed indicates that this model is adequate.

2. <u>Second Order Model of 4 Significant Variables</u>

The life predicting equation involving all first and second order terms of significant variables is obtained as

$$\begin{aligned}\hat{y} = 6.24 &- 0.255x_1 - 2.15x_2 + 0.0001x_5 - 0.377x_7 - 0.890x_1^2 \\ &+ 0.471x_2^2 - 0.0225x_5^2 - 0.260x_7^2 + 1.25x_1x_2 + 0.254x_1x_5 \\ &+ 0.128x_1x_7 - 1.39x_2x_5 + 1.28x_2x_7 - 0.270x_5x_7 \qquad (3)\end{aligned}$$

The analysis of variance of this equation is given in Table 5. The F-ratio was computed to be 59.02 with 14 and 3 degrees of freedom. The corresponding F value from the F-table is 8.71 at 95% significance level. The comparison of two F values indicates that the regression is effective and the model is acceptable. The residual sum of squares is 0.0353 in comparison to a total of 9.7843, a 1.4%. The other 99.6%

of the total is due to regression. Observing the sum of squares due to each individual term, it is obvious that for the first order terms the mean, variance and band width are significant. For the second order terms, the product of mean and variance, and the product of mean and duration of excursion above $\varepsilon_f$ level are significant. The individual effect of the duration of excursion above $\varepsilon_f$ level is found to be insignificant.

The predicted lives together with their residuals and 95% confidence intervals are given in Table 6. The confidence intervals here cover the actual fatigue lives of all tests including number 4. The reason is that the confidence intervals are fairly wide because of low degrees of freedom associated with t. The residuals are fairly random and are very small in magnitude. It can be considered a very good model.

## IV. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS AND SIX REPLICATIONS

This design consists of 24 tests for which three models have been developed. The first two are the first and second order models involving significant variables only. The third one involves first order terms of all 8 variables and second order terms of significant variables. These models are described below.

### 1. First Order Model of 4 Significant Variables

The first order life predicting equation of 4 significant variables is obtained as

$$\hat{y} = 5.82 - 0.056x_1 - 0.750x_2 - 0.0595x_6 - 0.0780x_7 \qquad (4)$$

The analysis of variance of this equation is given in Table 7. The F-ratio is computed to be 46.5 with 4 and 19 degrees of freedom which is greater than the corresponding F value from F-table of 2.90 at 95%

significance level. This comparison of two F values indicates that the regression is effective and the model is acceptable. The residual sum of squares is 1.2165 in comparison to a total of 13.1131, a 10.3%. The other 90.7% of the total is due to regression. The sum of squares due to the duration of excursion above $\varepsilon_f$ level is very low in comparison to the other three variables.

The predicted lives together with their residuals and 95% confidence interval of this equation are given in Table 8. The actual fatigue lives for test numbers 4, 13 and 17 fall out of the confidence interval. The confidence intervals are fairly narrow. The residuals appear to be randomly distributed.

2. Second Order Model of 4 Significant Variables

The second order life predicting equation using all first and second order terms of significant variables is obtained as

$$\begin{aligned}\hat{y} = 6.23 &+ 0.185x_1 - 1.88x_2 + 0.002x_6 - 0.310x_7 - 0.773x_1^2 \\ &+ 0.373x_2^2 - 0.0420x_6^2 - 0.248x_7^2 + 0.982x_1x_2 + 0.236x_1x_6 \\ &+ 0.178x_1x_7 - 1.10x_2x_6 + 1.16x_2x_7 - 0.267x_6x_7 \qquad (5)\end{aligned}$$

The analysis of variance of the equation above is given in Table 9. The F-ratio of 41.17 is obtained with 14 and 9 degrees of freedom. The corresponding F value from the F-table of 2.90 at 95% significance level is less than the F-ratio which implies that the regression is effective and the model is acceptable. The residual sum of squares is 0.2018 in comparison to a total of 13.1131, a 1.5%. The other 98.5% of the total is due to regression. Even though the duration of excursion above $\varepsilon_f$ level and its square are insignificant, its interactions with mean and band width are not insignificant.

The predicted life together with their residuals and 95% confidence intervals are given in Table 10. All confidence intervals cover the actual fatigue lives of all tests except for the test number 22 which is 0.7% below the lower level. The residuals are small and appear to be randomly distributed. The plot of residuals is given in Fig. 1.

3. Second Order Model Involving First Order Terms of All 8 Variables and Second Order Terms of 4 Significant Variables

The life predicting equation of this model is obtained as

$$\begin{aligned}\hat{y} = 6.09 &+ 0.182x_1 - 1.81x_2 + 0.0866x_3 + 0.0711x_4 + 0.172x_5 \\ &- 0.130x_6 - 0.370x_7 - 0.0487x_8 - 0.572x_1^2 + 0.408x_2^2 \\ &- 0.0510x_6^2 - 0.144x_7^2 + 0.969x_1x_2 + 0.194x_1x_6 + 0.161x_1x_7 \\ &- 1.15x_2x_6 + 0.936x_2x_7 - 0.226x_6x_7 \qquad (6)\end{aligned}$$

The analysis of variance of the above equation is given in Table 11. The F-ratio of 21.34 with 18 and 5 degrees of freedom is obtained for the above equation. The corresponding F value from the F-table at 95% significance level is 9.61 which is smaller than the F-ratio. Therefore, the regression is effective and the model is acceptable. The residual sum of squares is 0.1685 in comparison to a total of 13.1311, a 1.4%. The other 98.6% of the total is due to regression.

The predicted lives together with their residuals and 95% confidence intervals are given in Table 12. The residuals for all tests are very small. The low degrees of freedom associated with t makes the confidence intervals fairly wide. The actual lives of all tests fall within the predicted confidence intervals. The residuals appear to be randomly distributed.

On the basis of the analyses of variances of various models it is worth remarking that the duration of excursion above $\varepsilon_f$ level has an insignificant effect as an individual variable but its interactions with the other variables is not insignificant. It may also be observed that lower limit of confidence interval gives the safe estimate of fatigue life.

## V. DISCUSSIONS AND CONCLUSIONS

On the basis of the first order models obtained in [1], the best predicted lives and the lowest residual sum of squares are obtained for the model with all 8 variables. It is also found that there are only 4 variables which have significant effects on the fatigue life. These 4 variables are mean, variance, duration of excursion above $\varepsilon_f$ level, and the band width. In the present report, both first and second order models involving significant variables have been investigated.

In a comparative study of all 6 models investigated in this report and the best model of [1] as shown in Table 13, the second order model represented by equation (6) gives the best predicted lives and lowest residual sum of squares. From the analysis of this model it is observed that there are several terms which contribute a very small sum of squares to the regression. In the meantime they add one degree of freedom each to the regression. This leaves a less number of degrees of freedom associated with t which makes the 95% confidence intervals fairly wide. The residuals are very small because the number of variables regressed in this equation is relatively large considering the number of tests available.

The second order model represented by equation (5) has 4 terms less than those of the model represented by equation (6) but the change in the residual sum of squares is nominal as shown in Table 13. The predicted lives of equation (5) are also very close to actual lives. However, the confidence intervals are on an average 36.2% narrower than those of equation (6) as shown in Table 14.

Based on the overall consideration of the accuracy of the life prediction, residual sum of squares and the width of the confidence interval it may be concluded that the model of equation (5) is statistically better than the model of equation (6).

## VI. SUMMARY

(1) On the basis of analysis of first order models presented in [1], 4 variables showed significant effects on the fatigue life under random vibrations. These variables have been considered for further developing first and second order models in the present study.

(2) The first and second order models of significant variables have been developed separately for each of the three designs. In addition, one model involving first order terms of all 8 variables and second order terms of 4 significant variables is also obtained.

(3) For each model the analysis of variance and predicted lives together with residuals and 95 percent confidence intervals are obtained.

(4) Among the first order models in the present and the previous report [1], the best first order model is found to be the one which consists of all 8 variables based on 24 tests as it should be.

(5) Among all the second order models investigated in the present report and the best first order model, a second order model of significant variables is found to be the statistically best one.

ACKNOWLEDGMENTS

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Table 1. Analysis of Variance of 10 Tests
First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.87 - 0.094x_1 - 0.782x_2 - 0.051x_6 - 0.100x_7$$

| Source | Sum of Squares | Degrees of Freedom | Mean Square | F-Ratio |
|---|---|---|---|---|
| Due to Mean | 0.3337 | 1 | 0.3337 | |
| Due to Variance | 4.8717 | 1 | 4.8717 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level | 0.0206 | 1 | 0.0206 | |
| Due to Band Width | 0.1645 | 1 | 0.1645 | |
| Due to Regression | 5.3905 | 4 | 1.3476 | |
| Residuals | 0.3983 | 5 | 0.0797 | 16.91 |
| Total | 5.7888 | 9 | | |

F-ratio is greater than the table value of 5.19 with 4 and 5 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 2. Results of 10 Tests, First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.87 - 0.094x_1 - 0.782x_2 - 0.051x_6 - 0.100x_7$$

| Test No. | Actual Life | | Predicted Life | | Residuals | 95% Confidence Interval | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\hat{y}$ | | $\hat{T}$ | |
| | T | y | $\hat{y}$ | $\hat{T}$ | $y-\hat{y}$ | Lower | Upper | Lower | Upper |
| 1 | 1363.43 | 7.218 | 7.180 | 1312.91 | 0.038 | 6.640 | 7.720 | 765.16 | 2252.76 |
| 2 | 938.83 | 6.845 | 6.718 | 827.16 | 0.127 | 6.152 | 7.284 | 469.83 | 1456.25 |
| 3 | 165.08 | 5.106 | 5.402 | 221.85 | -0.296 | 4.960 | 5.844 | 142.56 | 345.23 |
| 4 | 391.97 | 5.971 | 6.339 | 566.23 | -0.367 | 5.858 | 6.820 | 350.10 | 915.78 |
| 5 | 156.20 | 5.051 | 5.171 | 176.09 | -0.120 | 4.634 | 5.708 | 102.89 | 301.37 |
| 6 | 160.83 | 5.080 | 4.796 | 121.03 | 0.284 | 4.254 | 5.338 | 70.35 | 208.20 |
| 7 | 1011.42 | 6.919 | 6.736 | 842.19 | 0.183 | 6.206 | 7.266 | 495.90 | 1430.28 |
| 8 | 259.08 | 5.557 | 5.434 | 229.06 | 0.123 | 4.992 | 5.876 | 147.20 | 356.46 |
| 9 | 347.50 | 5.851 | 5.920 | 372.41 | -0.069 | 5.617 | 6.223 | 274.96 | 504.40 |
| 10 | 370.33 | 5.914 | 5.817 | 335.96 | 0.097 | 5.148 | 6.539 | 172.18 | 691.91 |

Table 3. Analysis of Variance of 18 Tests
First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.84 - 0.039x_1 - 0.708x_2 - 0.080x_6 - 0.108x_7$$

| | Sum of Squares | Degrees of Freedom | Mean Square | F-Ratio |
|---|---|---|---|---|
| Due to Mean | 0.1212 | 1 | 0.1212 | |
| Due to Variance | 8.4720 | 1 | 8.4720 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level | 0.0398 | 1 | 0.0398 | |
| Due to Band Width | 0.2945 | 1 | 0.2945 | |
| Due to Regression | 8.9276 | 4 | 2.2319 | |
| Residuals | 0.8568 | 13 | 0.0659 | 33.86 |
| Total | 9.7843 | 17 | | |

F-ratio is greater than the table value of 3.18 with 4 and 13 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 4. Results of 18 Tests, First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.84 - 0.039x_1 - 0.708x_2 - 0.080x_6 - 0.108x_7$$

| Test No. | Actual Life | | Predicted Life | | Residuals | 95% Confidence Interval | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\hat{y}$ | | $\hat{T}$ | |
| | T | y | $\hat{y}$ | $\hat{T}$ | $y-\hat{y}$ | Lower | Upper | Lower | Upper |
| 1 | 1363.43 | 7.218 | 7.120 | 1236.45 | 0.097 | 6.760 | 7.480 | 863.83 | 1769.69 |
| 2 | 938.83 | 6.845 | 6.642 | 766.63 | 0.202 | 6.355 | 6.930 | 575.20 | 1021.76 |
| 3 | 165.08 | 5.106 | 5.212 | 183.46 | 0.175 | 4.834 | 5.590 | 125.71 | 267.76 |
| 4 | 391.97 | 5.971 | 6.320 | 555.57 | -0.349 | 6.000 | 6.640 | 403.56 | 764.85 |
| 5 | 156.20 | 5.051 | 5.191 | 179.65 | -0.140 | 4.841 | 5.541 | 126.61 | 254.91 |
| 6 | 160.83 | 5.080 | 4.896 | 133.75 | 0.180 | 4.615 | 5.177 | 101.01 | 177.12 |
| 7 | 1011.42 | 6.919 | 6.752 | 855.77 | 0.167 | 6.439 | 7.065 | 625.66 | 1170.52 |
| 8 | 259.08 | 5.557 | 5.458 | 234.63 | 0.100 | 5.143 | 5.773 | 171.17 | 321.62 |
| 9 | 347.50 | 5.851 | 5.937 | 378.80 | -0.087 | 5.790 | 6.084 | 327.05 | 438.73 |
| 10 | 370.33 | 5.914 | 5.755 | 315.77 | 0.159 | 5.461 | 6.049 | 235.39 | 423.59 |
| 11 | 346.00 | 5.846 | 5.892 | 362.13 | -0.046 | 5.756 | 6.028 | 316.06 | 414.92 |
| 12 | 371.00 | 5.916 | 5.918 | 371.67 | -0.001 | 5.778 | 6.058 | 322.98 | 427.69 |
| 13 | 467.83 | 6.148 | 5.869 | 353.89 | 0.279 | 5.530 | 6.208 | 252.11 | 496.77 |
| 14 | 407.33 | 6.010 | 5.978 | 394.65 | 0.031 | 5.665 | 6.291 | 288.53 | 539.80 |
| 15 | 98.67 | 4.592 | 4.528 | 92.57 | .064 | 4.221 | 4.835 | 68.12 | 125.80 |
| 16 | 1327.33 | 7.191 | 7.211 | 1354.24 | -0.020 | 6.924 | 7.498 | 1016.09 | 1804.94 |
| 17 | 189.08 | 5.242 | 5.910 | 368.70 | -0.668 | 5.668 | 6.152 | 289.47 | 469.62 |
| 18 | 273.67 | 5.612 | 5.480 | 239.85 | 0.132 | 5.186 | 5.774 | 178.79 | 321.74 |

Table 5. Analysis of Variance of 18 Tests
Second Order Model of 4 Significant Variable
Life predicting equation:

$$\hat{y} = 6.24 - 0.255x_1 - 2.15x_2 + 0.0001x_6 - 0.337x_7 - 0.890x_1^2 + 0.471x_2^2 - 0.0225x_6^2 - 0.260x_7^2 + 1.25x_1x_2 + 0.254x_1x_6 + 0.128x_1x_7 - 1.39x_2x_6 + 1.28x_2x_7 - 0.270x_6x_7$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|---|---|---|---|---|
| Due to Mean | 0.1212 | 1 | 0.1212 | |
| Due to Variance | 8.4720 | 1 | 8.4720 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level | 0.0398 | 1 | 0.0398 | |
| Due to Band Width | 0.2945 | 1 | 0.2945 | |
| Due to Mean Square | 0.0653 | 1 | 0.0653 | |
| Due to Variance Square | 0.0251 | 1 | 0.0251 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level Square | 0.0666 | 1 | 0.0666 | |
| Due to Band Width Square | 0.0115 | 1 | 0.0115 | |
| Due to Mean * Variance | 0.1948 | 1 | 0.1948 | |
| Due to Mean * Duration of Excursion Above $\varepsilon_f$ Level | 0.4194 | 1 | 0.4194 | |
| Due to Mean * Band Width | 0.0006 | 1 | 0.0006 | |
| Due to Variance * Band Width | 0.0030 | 1 | 0.0030 | |
| Due to Variance * Duration of Excursion | 0.0070 | 1 | 0.0070 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level * Band Width | 0.0281 | 1 | 0.0281 | |
| Due to Regression | 9.7490 | 14 | 0.6964 | |
| Residuals | 0.0353 | 3 | 0.0118 | 59.02 |
| Total | 9.7843 | | | |

F-ratio is greater than the table value 8.71 with 14 and 3 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 6. **Results of 18 Tests, Second Order Model of 4 Significant Variables**
**Life predicting equation:**

$$y = 6.24 - 0.255x_1 - 2.15x_2 + 0.0001x_6 - 0.337x_7 - 0.190x_1^2 + 0.471x_2^2$$
$$- 0.0225x_6^2 - 0.260x_7^2 + 1.25x_1x_2 + 0.254x_1x_6 + 0.128x_1x_7 - 1.39x_2x_6$$
$$+ 1.28x_2x_7 - 0.270x_6x_7$$

| Test No. | Actual Life | | Predicted Life | | Residuals | 95% Confidence Interval | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\hat{y}$ | | $\hat{T}$ | |
| | T | y | $\hat{y}$ | $\hat{T}$ | $y-\hat{y}$ | Lower | Upper | Lower | Upper |
| 1 | 1363.43 | 7.218 | 7.243 | 1398.28 | -0.025 | 6.903 | 7.583 | 994.78 | 1965.45 |
| 2 | 938.83 | 6.845 | 6.888 | 980.44 | -0.044 | 6.554 | 7.222 | 701.97 | 1369.37 |
| 3 | 165.08 | 5.106 | 5.091 | 162.55 | 0.016 | 4.747 | 5.435 | 115.28 | 229.21 |
| 4 | 391.97 | 5.971 | 6.062 | 429.23 | -0.091 | 5.763 | 6.361 | 318.27 | 578.89 |
| 5 | 156.20 | 5.051 | 5.050 | 156.02 | 0.001 | 4.703 | 5.397 | 110.29 | 220.70 |
| 6 | 160.83 | 5.080 | 5.082 | 161.09 | -0.002 | 4.738 | 5.426 | 114.24 | 227.16 |
| 7 | 1011.42 | 6.919 | 6.944 | 1036.91 | -0.025 | 6.600 | 7.288 | 735.35 | 1462.14 |
| 8 | 259.08 | 5.557 | 5.526 | 251.14 | 0.031 | 5.186 | 5.866 | 178.67 | 353.00 |
| 9 | 347.50 | 5.851 | 5.866 | 352.83 | -0.015 | 5.675 | 6.057 | 291.52 | 427.06 |
| 10 | 370.23 | 5.914 | 5.867 | 353.19 | 0.047 | 5.533 | 6.201 | 252.87 | 493.30 |
| 11 | 346.00 | 5.846 | 5.911 | 369.08 | -0.064 | 5.714 | 6.108 | 302.99 | 449.57 |
| 12 | 371.00 | 5.916 | 5.913 | 369.81 | 0.004 | 5.709 | 6.167 | 301.57 | 453.34 |
| 13 | 467.83 | 6.148 | 6.097 | 444.52 | 0.051 | 5.767 | 6.428 | 319.28 | 618.89 |
| 14 | 407.33 | 6.010 | 6.015 | 409.53 | -0.005 | 5.671 | 6.359 | 290.42 | 577.47 |
| 15 | 98.67 | 4.592 | 4.634 | 102.92 | -0.042 | 4.300 | 4.968 | 73.69 | 143.75 |
| 16 | 1327.33 | 7.191 | 7.100 | 1211.96 | 0.091 | 6.801 | 7.399 | 898.65 | 1634.52 |
| 17 | 189.08 | 5.242 | 5.226 | 186.05 | 0.016 | 4.882 | 5.570 | 131.94 | 262.34 |
| 18 | 273.67 | 5.612 | 5.556 | 258.79 | 0.056 | 5.228 | 5.884 | 186.47 | 359.15 |

Table 7. Analysis of Variance of 24 Tests
First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.82 - 0.0656x_1 - 0.750x_2 - 0.0595x_6 - 0.0780x_7$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|---|---|---|---|---|
| Due to Mean | 0.1172 | 1 | 0.1172 | |
| Due to Variance | 11.5157 | 1 | 11.5157 | |
| Due to Duration of Excursion Above $\varepsilon_f$ | 0.0305 | 1 | 0.0305 | |
| Due to Band Width | 0.2331 | 1 | 0.2331 | |
| Due to Regression | 11.8966 | 4 | 2.9742 | |
| Residuals | 1.2165 | 19 | 0.0640 | 46.57 |
| Total | 13.1131 | | | |

F-ratio is greater than the table value 2.90 with 4 and 19 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 8. Results of 24 Tests, First Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 5.82 - 0.0656x_1 - 0.750x_2 - 0.0595x_6 - 0.0780x_7$$

| Test No. | Actual Life | | Predicted Life | | Residuals | 95% Confidence Interval | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\hat{y}$ | | $\hat{T}$ | |
| | T | y | $\hat{y}$ | $\hat{T}$ | $y-\hat{y}$ | Lower | Upper | Lower | Upper |
| 1 | 1363.43 | 7.218 | 7.057 | 1160.95 | 0.161 | 6.751 | 7.363 | 855.27 | 1575.89 |
| 2 | 938.83 | 6.845 | 6.674 | 791.56 | 0.171 | 6.431 | 6.917 | 620.93 | 1009.07 |
| 3 | 165.08 | 5.106 | 5.163 | 174.69 | -0.056 | 4.820 | 5.506 | 123.93 | 246.23 |
| 4 | 391.47 | 5.971 | 6.359 | 577.67 | -0.388 | 6.108 | 6.610 | 449.37 | 742.60 |
| 5 | 156.20 | 5.051 | 5.063 | 158.06 | -0.011 | 4.768 | 5.358 | 117.67 | 212.32 |
| 6 | 160.83 | 5.080 | 4.838 | 126.22 | 0.242 | 4.543 | 5.133 | 93.96 | 169.54 |
| 7 | 1011.42 | 6.919 | 6.705 | 816.48 | 0.215 | 6.437 | 6.973 | 624.59 | 1067.32 |
| 8 | 259.08 | 5.557 | 5.332 | 206.85 | 0.225 | 5.093 | 5.571 | 162.94 | 262.59 |
| 9 | 347.50 | 5.851 | 5.887 | 360.32 | -0.036 | 5.761 | 6.013 | 317.80 | 408.54 |
| 10 | 370.33 | 5.914 | 5.760 | 317.35 | 0.154 | 5.500 | 6.019 | 244.81 | 411.39 |
| 11 | 346.00 | 5.846 | 5.850 | 347.23 | -0.003 | 5.737 | 5.963 | 310.13 | 388.78 |
| 12 | 371.00 | 5.916 | 5.864 | 352.13 | 0.052 | 5.745 | 6.035 | 312.53 | 417.92 |
| 13 | 467.83 | 6.148 | 5.791 | 327.34 | 0.357 | 5.502 | 6.080 | 245.22 | 436.96 |
| 14 | 407.33 | 6.010 | 5.984 | 397.02 | 0.026 | 5.743 | 6.225 | 312.09 | 505.06 |
| 15 | 98.67 | 4.592 | 4.418 | 82.93 | 0.173 | 4.175 | 4.661 | 65.05 | 105.72 |
| 16 | 1327.33 | 7.191 | 7.239 | 1392.70 | -0.048 | 6.984 | 7.494 | 1078.85 | 1797.85 |
| 17 | 189.08 | 5.242 | 5.849 | 346.89 | -0.607 | 5.633 | 6.065 | 279.62 | 430.34 |
| 18 | 273.67 | 5.612 | 5.497 | 243.96 | 0.115 | 5.273 | 5.721 | 195.01 | 305.19 |
| 19 | 430.00 | 6.064 | 5.929 | 375.78 | 0.135 | 5.703 | 6.155 | 299.75 | 471.09 |
| 20 | 157.17 | 5.057 | 5.104 | 164.68 | -0.046 | 4.870 | 5.338 | 130.27 | 208.18 |
| 21 | 127.00 | 4.844 | 4.871 | 130.45 | -0.027 | 4.632 | 5.110 | 102.76 | 165.60 |
| 22 | 137.42 | 4.923 | 5.359 | 212.51 | -0.436 | 5.120 | 5.597 | 167.40 | 269.78 |
| 23 | 484.42 | 6.183 | 6.320 | 555.57 | -0.138 | 6.077 | 6.563 | 435.81 | 708.24 |
| 24 | 136.75 | 4.918 | 5.147 | 171.91 | -0.228 | 4.904 | 5.390 | 134.86 | 219.16 |

Table 9. Analysis of Variance of 24 Tests
Second Order Model of 4 Significant Variables
Life predicting equation:

$$\hat{y} = 6.23 + 0.185x_1 - 1.88x_2 + 0.0002x_6 - 0.310x_7 - 0.773x_1^2 + 0.373x_2^2 - 0.042x_6^2 - 0.248x_7^2 + 0.982x_1x_2 + 0.236x_1x_6 + 0.178x_1x_7 - 1.10x_2x_6 - 1.116x_2x_7 - 0.267x_6x_7$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|---|---|---|---|---|
| Due to Mean | 0.1172 | 1 | 0.1172 | |
| Due to Variance | 11.5157 | 1 | 11.5157 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level | 0.0305 | 1 | 0.0305 | |
| Due to Band Width | 0.2331 | 1 | 0.2331 | |
| Due to Mean Square | 0.0633 | 1 | 0.0633 | |
| Due to Variance Square | 0.0240 | 1 | 0.0240 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Level Square | 0.0004 | 1 | 0.0004 | |
| Due to Band Width Square | 0.0025 | 1 | 0.0025 | |
| Due to Mean * Variance | 0.1252 | 1 | 0.1252 | |
| Due to Mean * Duration of Excursion Above $\varepsilon_f$ Level | 0.5617 | 1 | 0.5617 | |
| Due to Mean * Band Width | 0.0013 | 1 | 0.0013 | |
| Due to Variance * Duration of Excursion Above $\varepsilon_f$ Level | 0.0200 | 1 | 0.0200 | |
| Due to Variance * Band Width | 0.1084 | 1 | 0.1084 | |
| Due to Band Width * Duration of Excursion Above $\varepsilon_f$ | 0.1078 | 1 | 0.1078 | |
| Due to Regression | 12.9113 | 14 | 0.9222 | |
| Residuals | 0.2018 | 9 | 0.0224 | 41.17 |
| Total | 13.1131 | 23 | | |

F-ratio is greater than the table value 2.90 with 14 and 9 degrees of freedom at 95% significance level. So the regression is effecitve and the model is accepted.

**Table 10. Results of 24 Tests, Second Order Model of 4 Significant Variables. Life predicting equation:**

$$y = 6.23 + 0.185x_1 - 1.88x_2 + 0.0002x_6 - 0.310x_7 - 0.773x_1^2 + 0.373x_2^2 - 0.0420x_6^2$$
$$- 0.248x_7^2 + 0.982x_1x_2 + 0.238x_1x_6 + 0.178x_1x_7 - 1.10x_2x_6 + 1.16x_2x_7 - 0.267x_6x_7$$

| Test No. | Actual Life | | Predicting Life | | Residuals | 95% Confidence Interval | | | |
|---|---|---|---|---|---|---|---|---|---|
| | T | y | $\hat{y}$ | $\hat{T}$ | $y-\hat{y}$ | $\hat{y}$ Lower | $\hat{y}$ Upper | $\hat{T}$ Lower | $\hat{T}$ Upper |
| 1 | 1363.43 | 7.218 | 7.238 | 1391.31 | -0.021 | 6.910 | 7.566 | 1002.26 | 1931.38 |
| 2 | 938.83 | 6.845 | 6.883 | 975.55 | -0.039 | 6.591 | 7.175 | 728.66 | 1306.09 |
| 3 | 165.08 | 5.106 | 5.105 | 164.84 | 0.001 | 4.845 | 5.365 | 127.09 | 213.82 |
| 4 | 391.97 | 5.971 | 5.979 | 395.05 | -0.008 | 5.741 | 6.216 | 311.53 | 500.95 |
| 5 | 156.20 | 5.051 | 5.034 | 153.55 | 0.017 | 4.713 | 5.355 | 111.36 | 211.71 |
| 6 | 160.83 | 5.080 | 5.093 | 162.88 | -0.013 | 4.871 | 5.315 | 130.50 | 203.30 |
| 7 | 1011.42 | 6.919 | 6.954 | 1047.33 | -0.035 | 6.732 | 7.284 | 839.09 | 1457.17 |
| 8 | 259.08 | 5.557 | 5.328 | 206.03 | 0.229 | 5.156 | 5.500 | 173.48 | 244.67 |
| 9 | 347.50 | 5.851 | 5.868 | 353.54 | -0.017 | 5.696 | 6.032 | 297.70 | 416.51 |
| 10 | 370.23 | 5.914 | 5.863 | 351.78 | 0.051 | 5.605 | 6.121 | 271.82 | 455.26 |
| 11 | 346.00 | 5.846 | 5.910 | 368.71 | -0.064 | 5.724 | 6.095 | 306.28 | 443.85 |
| 12 | 371.00 | 5.916 | 5.907 | 367.60 | 0.009 | 5.722 | 6.092 | 305.37 | 442.52 |
| 13 | 467.83 | 6.148 | 6.090 | 441.42 | 0.058 | 5.780 | 6.400 | 323.79 | 601.78 |
| 14 | 407.33 | 6.010 | 6.167 | 476.75 | -0.158 | 5.927 | 6.407 | 375.11 | 605.93 |
| 15 | 98.67 | 4.592 | 4.621 | 101.60 | -0.029 | 4.336 | 4.906 | 76.40 | 135.10 |
| 16 | 1327.33 | 7.191 | 7.106 | 1219.26 | 0.085 | 6.821 | 7.391 | 916.89 | 1621.35 |
| 17 | 189.08 | 5.242 | 5.245 | 189.62 | -0.002 | 4.917 | 5.573 | 136.59 | 263.22 |
| 18 | 273.67 | 5.612 | 5.525 | 250.89 | 0.087 | 5.525 | 5.794 | 250.88 | 328.83 |
| 19 | 430.00 | 6.064 | 5.885 | 359.60 | 0.179 | 5.632 | 6.138 | 288.07 | 463.28 |
| 20 | 157.17 | 5.057 | 5.051 | 156.18 | 0.007 | 4.741 | 5.361 | 114.56 | 212.92 |
| 21 | 127.00 | 4.844 | 4.862 | 129.28 | -0.018 | 4.620 | 5.104 | 101.49 | 164.68 |
| 22 | 137.42 | 4.923 | 5.159 | 173.99 | -0.236 | 4.931 | 5.747 | 138.45 | 313.39 |
| 23 | 484.42 | 6.183 | 6.262 | 524.27 | -0.079 | 6.038 | 6.486 | 419.08 | 655.85 |
| 24 | 136.75 | 4.918 | 4.924 | 137.55 | -0.006 | 4.600 | 5.247 | 99.54 | 190.08 |

Table 11. Analysis of Variance of 24 Tests
Second Order Model of All 8 Variables
Life predicting equation:

$$\hat{y} = 6.09 + 0.182x_1 - 1.81x_2 + 0.087x_3 + 0.071x_4 + 0.172x_5 - 0.130x_6 - 0.370x_7 - 0.049x_8 - 0.572x_1^2 + 0.408x_2^2 - 0.051x_6^2 - 0.144x_7^2 + 0.969x_1x_2 + 0.194x_1x_6 + 0.161x_1x_7 - 1.15x_2x_6 + 0.936x_2x_7 - 0.226x_6x_7$$

| Source | Sum of Squares | Degrees of Freedom | Mean Squares | F-Ratio |
|---|---|---|---|---|
| Due to Mean | 0.1172 | 1 | 0.1172 | |
| Due to Variance | 11.5157 | 1 | 11.5157 | |
| Due to Zero Upcrossings | 0.0021 | 1 | 0.0021 | |
| Due to $\varepsilon_f$ Level Upcrossings | 0.0708 | 1 | 0.0708 | |
| Due to Duration of Excursion Above Zero | 0.0554 | 1 | 0.0554 | |
| Due to Duration of Excursion Above $\varepsilon_f$ | 0.0041 | 1 | 0.0041 | |
| Due to Band Width | 0.6993 | 1 | 0.6993 | |
| Due to Average Amplitude Above $\varepsilon_f$ | 0.0015 | 1 | 0.0015 | |
| Due to Mean Square | 0.0009 | 1 | 0.0009 | |
| Due to Variance Square | 0.0417 | 1 | 0.0417 | |
| Due to Duration of Excursion Above $\varepsilon_f$ Square | 0.0286 | 1 | 0.0286 | |
| Due to Band Width Square | 0.0427 | 1 | 0.0427 | |
| Due to Mean * Variance | 0.0047 | 1 | 0.0047 | |
| Due to Mean * Duration of Excursion Above $\varepsilon_f$ | 0.0927 | 1 | 0.0927 | |
| Due to Mean * Band Width | 0.0223 | 1 | 0.0223 | |
| Due to Variance * Duration of Excursion Above $\varepsilon_f$ | 0.1095 | 1 | 0.1095 | |
| Due to Variance * Band Width | 0.0873 | 1 | 0.0873 | |
| Due to Band Width * Duration of Excursion Above $\varepsilon_f$ | 0.0480 | 1 | 0.0480 | |
| Due to Regression | 12.9446 | 18 | 0.7191 | |
| Residuals | 0.1685 | 5 | 0.0337 | 21.34 |
| Total | 13.1311 | 23 | | |

F-ratio is greater than the table value 9.61 with 18 and 5 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 12. Results of 24 Tests, Second Order Model of All 8 Variables. Life predicting equation:

$$\hat{y} = 6.09 + 0.182x_1 - 1.81x^2 + 0.087x_3 + 0.071x_4 + 0.172x_5 - 0.130x_6 - 0.370x_7 - 0.049x_8$$
$$- 0.572x_1^2 + 0.408x_2^2 - 0.057x_6^2 - 0.144x_7^2 + 0.969x_1x_2 + 0.194x_1x_6 + 0.161x_1x_7$$
$$- 1.15x_2x_6 + 0.936x_2x_7 - 0.266x_6x_7$$

| Test No. | Actual Life | | Predicted Life | | Residuals | 95% Confidence Interval | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $\hat{y}$ | | $\hat{T}$ | |
| | T | y | $\hat{y}$ | $\hat{T}$ | $y-\hat{y}$ | Lower | Upper | Lower | Upper |
| 1 | 1363.43 | 7.218 | 7.231 | 1381.60 | -0.013 | 6.768 | 7.694 | 869.76 | 2194.65 |
| 2 | 938.83 | 6.845 | 6.901 | 993.27 | -0.056 | 6.450 | 7.352 | 632.99 | 1558.60 |
| 3 | 165.08 | 5.106 | 5.053 | 156.49 | 0.053 | 4.610 | 5.496 | 100.48 | 243.71 |
| 4 | 391.97 | 5.971 | 6.005 | 405.45 | -0.034 | 5.658 | 6.352 | 286.48 | 573.84 |
| 5 | 156.20 | 5.051 | 5.042 | 154.78 | 0.009 | 4.581 | 5.503 | 97.65 | 245.33 |
| 6 | 160.83 | 5.080 | 5.171 | 176.09 | -0.090 | 4.801 | 5.541 | 121.63 | 254.93 |
| 7 | 1011.42 | 6.919 | 6.962 | 1055.75 | -0.043 | 6.506 | 7.417 | 669.43 | 1664.99 |
| 8 | 259.08 | 5.570 | 5.387 | 218.55 | 0.171 | 4.999 | 5.775 | 148.32 | 322.02 |
| 9 | 347.50 | 5.851 | 5.798 | 329.64 | 0.053 | 5.380 | 6.216 | 217.06 | 500.61 |
| 10 | 370.23 | 5.914 | 5.897 | 363.94 | 0.018 | 5.436 | 6.358 | 229.61 | 576.87 |
| 11 | 346.00 | 5.846 | 5.888 | 360.68 | -0.042 | 5.588 | 6.188 | 267.33 | 486.64 |
| 12 | 371.00 | 5.916 | 5.959 | 372.04 | -0.043 | 5.634 | 6.284 | 279.86 | 535.76 |
| 13 | 467.83 | 6.148 | 6.063 | 429.66 | 0.085 | 5.612 | 6.514 | 273.82 | 674.21 |
| 14 | 407.33 | 6.010 | 6.116 | 453.05 | -0.106 | 5.723 | 6.509 | 305.93 | 670.92 |
| 15 | 98.67 | 4.592 | 4.590 | 98.49 | 0.002 | 4.185 | 4.995 | 65.68 | 147.71 |
| 16 | 1327.33 | 7.191 | 7.101 | 1213.18 | 0.090 | 6.701 | 7.501 | 813.05 | 1810.22 |
| 17 | 189.08 | 5.242 | 5.245 | 189.62 | -0.003 | 4.784 | 5.706 | 119.63 | 300.55 |
| 18 | 273.67 | 5.612 | 5.491 | 242.50 | 0.121 | 5.030 | 5.914 | 152.99 | 370.13 |
| 19 | 430.00 | 6.064 | 5.929 | 375.78 | 0.135 | 5.564 | 6.294 | 260.87 | 541.30 |
| 20 | 157.17 | 5.057 | 5.093 | 162.88 | -0.036 | 4.647 | 5.539 | 104.32 | 254.30 |
| 21 | 127.00 | 4.844 | 4.828 | 124.96 | 0.016 | 4.500 | 5.206 | 85.6[illegible] | 182.28 |
| 22 | 137.42 | 4.923 | 5.146 | 1[illegible]1.74 | -0.223 | 4.766 | 5.526 | 117.44 | 215.15 |
| 23 | 484.42 | 6.183 | 6.251 | 518.53 | -0.069 | 5.876 | 6.626 | 356.37 | 754.48 |
| 24 | 136.75 | 4.918 | 4.914 | 136.18 | 0.004 | 4.453 | 5.375 | 85.92 | 215.86 |

Table 13 Comparison of Percent Deviations of Predicted Lives and Residual Sum of Squares for Seven Models

| Test No. | Actual Life T | Percent Deviations of Predicted Lives | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Eq(1) | Eq(2) | Eq(3) | Eq(4) | Eq(5) | Eq(6) | Eq(6) of [1] |
| 1 | 1363.43 | 3.7 | 9.3 | -2.6 | 14.8 | -2.1 | -1.3 | 15.9 |
| 2 | 938.83 | 11.9 | 18.3 | -4.4 | 15.7 | -3.9 | 5.8 | 14.6 |
| 3 | 165.08 | -25.6 | -11.1 | 1.5 | -5.8 | 0.2 | 5.2 | -23.1 |
| 4 | 391.97 | -44.5 | -41.7 | -9.5 | -47.4 | -0.8 | 3.4 | -59.7 |
| 5 | 156.20 | -12.7 | -15.0 | 0.1 | -1.2 | 1.7 | 0.9 | -1.4 |
| 6 | 160.83 | 24.7 | 16.8 | -0.1 | 21.5 | -1.3 | 9.5 | 20.8 |
| 7 | 1011.42 | 16.7 | 15.4 | -2.5 | 19.3 | -3.6 | -4.4 | 19.8 |
| 8 | 259.08 | 11.6 | 9.4 | 3.1 | 20.2 | 20.4 | 15.6 | 24.1 |
| 9 | 347.50 | -7.2 | -9.0 | -1.5 | -3.7 | -1.7 | 5.1 | -10.4 |
| 10 | 370.23 | 9.3 | 14.7 | 4.6 | 14.3 | 5.0 | 1.7 | 16.2 |
| 11 | 346.00 | | -4.7 | -6.7 | -0.4 | -6.6 | -4.2 | -2.7 |
| 12 | 371.00 | | -0.2 | 0.3 | 5.1 | 0.9 | -0.3 | 8.4 |
| 13 | 467.82 | | 24.4 | 5.0 | 30.0 | 0.9 | 8.2 | 20.2 |
| 14 | 407.33 | | 3.1 | -0.5 | 2.5 | -17.0 | -11.2 | 6.3 |
| 15 | 98.67 | | 6.2 | -4.3 | 15.9 | -3.0 | 0.2 | 15.5 |
| 16 | 1327.33 | | -2.0 | 8.7 | -4.9 | 8.1 | 8.6 | 3.7 |
| 17 | 189.08 | | -95.0 | 1.6 | -83.5 | 0.3 | -0.3 | 6.8 |
| 18 | 273.67 | | 12.4 | 5.4 | 10.9 | 8.3 | 11.4 | 16.3 |
| 19 | 430.00 | | | | 12.6 | 16.4 | 12.6 | 4.3 |
| 20 | 152.17 | | | | -4.8 | -2.6 | -0.4 | -2.4 |
| 21 | 127.00 | | | | -2.7 | -1.8 | 1.6 | -7.8 |
| 22 | 137.42 | | | | -54.6 | -26.6 | -24.9 | -55.9 |
| 23 | 484.42 | | | | -14.7 | 8.2 | -7.0 | -20.7 |
| 24 | 136.75 | | | | -25.7 | -0.6 | 0.4 | -25.7 |

| | Average Deviations | | | | | | |
|---|---|---|---|---|---|---|---|
| Negative side | 22.5 | 22.3 | 3.6 | 20.8 | 5.3 | 6.0 | 17.4 |
| Positive side | 13.0 | 13.0 | 3.4 | 15.2 | 6.8 | 6.0 | 16.0 |
| | **Residuals** | | | | | | |
| Percent residual sum of squares of the total | 17.0 | 8.8 | 0.4 | 9.3 | 1.5 | 1.3 | 7.3 |

Table 14 Comparison of Widths of the Confidence Intervals for Two Models

| Test No. | Actual Life T | Models: Equation (5) Predicted Life T | Equation (5) Lower Limit L | Equation (5) Upper Limit U | Equation (6) Predicted Life T | Equation (6) Lower Limit L | Equation (6) Upper Limit U | Percent Ratio $\frac{(U-L)_5}{(U-L)_6} \times 100$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1363.43 | 1391.31 | 1002.26 | 1931.38 | 1381.60 | 869.78 | 2164.65 | 71.7 |
| 2 | 938.83 | 975.55 | 728.66 | 1306.09 | 993.27 | 632.99 | 1558.60 | 62.4 |
| 3 | 165.08 | 164.83 | 127.09 | 213.82 | 156.49 | 100.48 | 243.71 | 60.6 |
| 4 | 391.97 | 395.05 | 311.53 | 500.95 | 405.45 | 286.48 | 573.84 | 65.9 |
| 5 | 156.20 | 153.55 | 111.36 | 211.71 | 154.78 | 97.65 | 245.33 | 68.0 |
| 6 | 160.83 | 162.88 | 130.50 | 203.30 | 176.09 | 121.63 | 254.93 | 54.6 |
| 7 | 1011.42 | 1047.33 | 839.09 | 1457.17 | 1055.75 | 669.43 | 1664.99 | 62.1 |
| 8 | 259.08 | 206.03 | 173.48 | 244.67 | 218.55 | 148.32 | 322.02 | 41.0 |
| 9 | 347.50 | 353.54 | 297.70 | 416.51 | 329.64 | 217.06 | 500.61 | 42.0 |
| 10 | 370.23 | 351.78 | 271.82 | 455.26 | 363.94 | 229.61 | 576.87 | 52.8 |
| 11 | 346.00 | 368.71 | 306.28 | 443.85 | 360.68 | 263.33 | 486.64 | 61.6 |
| 12 | 371.00 | 367.70 | 305.37 | 442.52 | 372.04 | 279.86 | 535.76 | 53.6 |
| 13 | 467.83 | 441.42 | 323.79 | 601.78 | 429.66 | 273.82 | 674.21 | 69.4 |
| 14 | 407.33 | 476.75 | 375.11 | 605.93 | 453.05 | 305.93 | 670.92 | 63.2 |
| 15 | 98.67 | 101.60 | 76.40 | 135.10 | 98.49 | 65.68 | 147.71 | 71.6 |
| 16 | 1327.33 | 1219.26 | 916.89 | 1621.35 | 1213.18 | 813.05 | 1810.22 | 70.6 |
| 17 | 189.08 | 189.62 | 136.59 | 263.22 | 189.62 | 119.63 | 300.55 | 70.1 |
| 18 | 273.67 | 250.89 | 250.88 | 328.83 | 242.50 | 152.99 | 370.13 | 35.9 |
| 19 | 430.00 | 359.60 | 288.07 | 463.28 | 375.78 | 260.87 | 541.30 | 62.5 |
| 20 | 157.17 | 156.18 | 114.56 | 212.92 | 162.88 | 104.32 | 254.30 | 65.6 |
| 21 | 127.00 | 129.28 | 101.49 | 164.68 | 124.96 | 85.67 | 182.28 | 65.4 |
| 22 | 137.42 | 173.99 | 138.45 | 313.39 | 171.74 | 117.44 | 251.14 | 130.8 |
| 23 | 484.42 | 524.27 | 419.08 | 655.85 | 518.53 | 356.37 | 754.48 | 59.5 |
| 24 | 136.75 | 137.55 | 99.54 | 190.08 | 136.18 | 85.92 | 215.86 | 69.6 |
| Average: | | | | | | | | 63.8% |

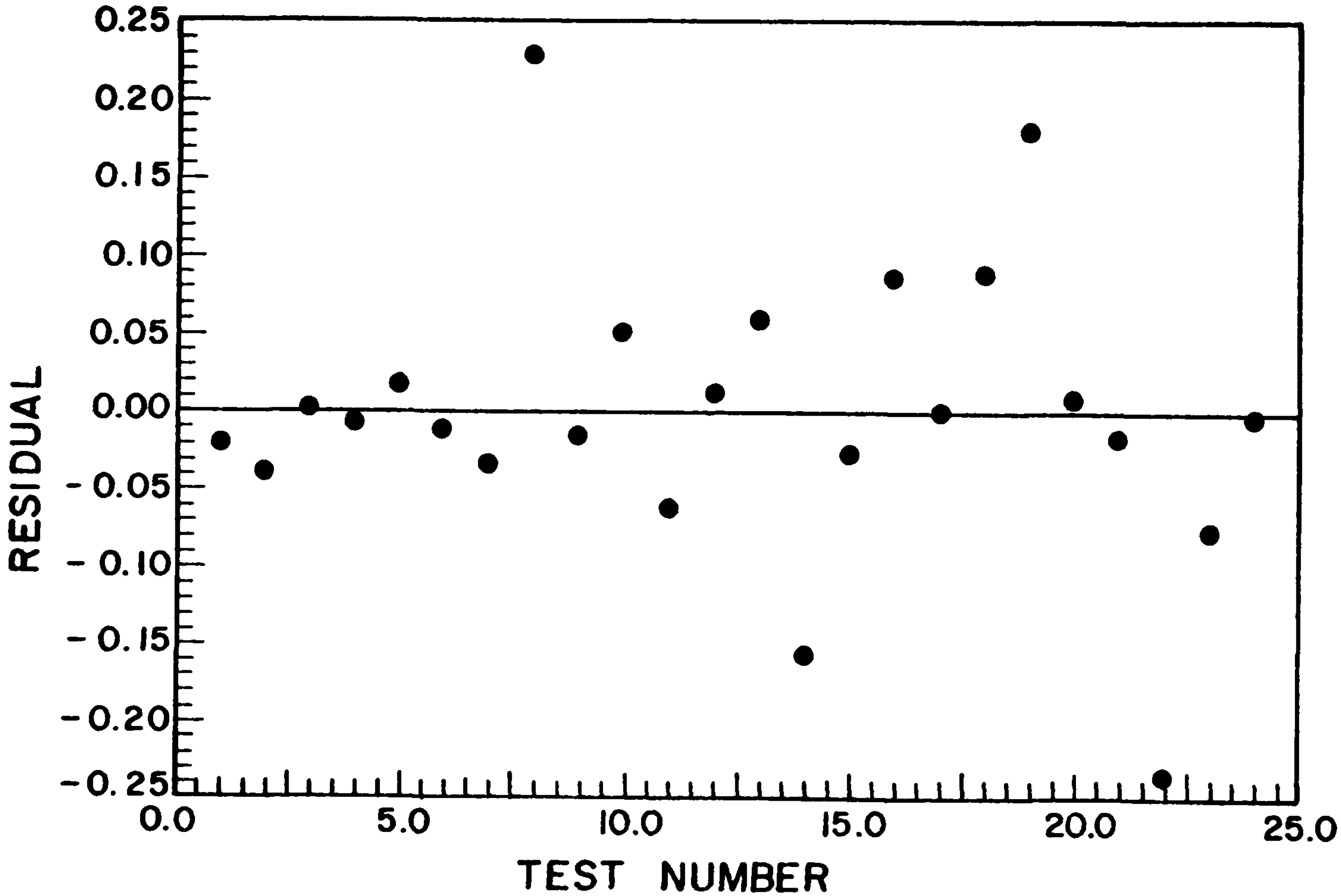


Fig. 1 Distribution of Residuals for the Best Second Order Model, Eq. (5)

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In the previous study of first order models with 8 response variables to predict fatigue life in elastic range under random vibrations, 4 variables showed significant effects. In this part of the research both first and second order models of these 4 significant variables based on 10, 18, and 24 tests are developed. The tables of analysis of variance, and of predicted lives together with their residuals and 95% confidence intervals are constructed for each of the first and second order models.

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